American Nuclear Society Annual Meeting June 2023, Indianapolis
Transactions of the ANS, Vol.128, pp.617-620.
With editorial corrections, Rev.24.

Verified Knowledge of Nuclear Power Plants Using the NCV Method

Fred D. Lang, P.E., P.Eng.

Exergetic Systems Limited
Salt Spring Island, British Columbia, Canada
Lang@ExergeticSystems.com

INTRODUCTION

At the invitation of a nuclear utility, the Neutronic, Calorimetric & Verification (NCV) Method was developed.[1] The goal was to improve NSSS safety. Prior experiences with creating large system simulators (PEPSE, THERM, EX-SITE, EX-FOSS, INPUT/LOSS, etc.), if using a closed solution, meant first one must choose viable unknowns, and only then develop a non-sparse set of equations. Declared unknowns consisted of the major actors: reactor vessel coolant flow, absolute neutron flux and condenser heat rejection. However, a fourth "unknown" is included, gross electrical generation (explained below). A goal was to establish a nexus between average neutron flux and condenser heat rejection; a nexus between main coolant flow and generation; etc. This approach to improve Nuclear Steam Supply System (NSSS) safety.

The NCV Method treats fission as an inertial process. This means assuming a phenomenon which is self-contained following incident neutron capture. It recognizes that the exergy released from fission has no thermodynamic reference. We would measure the same MeV/Fission in deep space or under miles of ocean. Using an Δenthalpy across the reactor vessel times an assumed coolant flow, *per se*, makes little sense if we wish to couple neutron flux to generation, etc. Given an inertial process, the only viable engineering tool which allows resolution lies with the Second Law. Writing a Second Law equation also means that irreversible losses (antineutrino, Carnot engine losses, total heat exchanger Δexergy, etc.) must be correctly treated.

PAPER-92.WPD
Revision 24, May 31, 2023

EQUATION SET

Thus the governing equation is a Second Law balance across the complete NSSS. This equation, with additional two First Law and another Second, resolves the four unknowns. From this closed solution, not only are the unknowns resolved, but the solution is then verified. In summary, the NCV Method is based on routine **N**eutronics, First & Second Law **C**alorimetrics and optimization procedures producing demonstrable **V**erification. *Prima facie* verification is accomplished using electrical generation: NCV compares the computed to the directly measured at the terminals, and then reduces differences using established optimization routines. This process has <u>no</u> affect on the thermodynamic balances. Governing equation follows, using abbreviated nomenclature:

$$A_1 [\bar{\nu}_{REC}(t) + \bar{\nu}_{LRV}(t)]\, \Phi_{TH} - P_{GEN} + A_3 Q_{REJ} + A_4 m_{RV} = L_A + A_1 \psi_{LRV} \qquad (2ND')$$

Eq.(2ND′) is unique especially in its treatment of the antineutrino $\bar{\nu}_{LRV}$ term. It, of course, appears on the left-side as a portion of the total exergy produced from fission. However, instead of cancelling with its irreversible term on the right, a ψ_{LRV} term is introduced defined as $[\bar{\nu}_{LRV}\Phi_{TH}]$. ψ_{LRV} may be defined as a constant, or may be determined independently using the verification procedure.

A First Law balance about the Turbine Cycle (TC) is routine ... at least at first blush:

$$- P_{GEN} - Q_{REJ} + C_4 m_{RV} = L_C \qquad (TC')$$

The C_4 coefficient consists of both a Δh term for feedwater flow (converted to reactor flow for the PWR) and pump terms. Losses, L_C, include miscellaneous vessel casing losses [turbine, feedwater heaters (FW), MSR, etc.], credit for an auxiliary turbine driven FW pump, generator losses, etc. Note that extraneous TC losses could be selected as a Choice Operating Parameter (COP, discussed below).

There remains two independent equations required. The first is a First Law balance about the complete NSSS. The overt concern about proper treatment of the inertial process - applicable for Second Law analysis - cannot thwart use of a viable First Law equation. Thus created was an "Inertial Conversion Factor", $\Xi(T_{Ref})$, which converts the Δexergy to a Δenthalpy. T_{Ref} is defined through iterative procedures using exergy's given definition: $g = f(P,h,s,T_{Ref})$. Typically, $\Xi(T_{Ref})$ is resolved by balancing Eq.(1ST′), a converted Δexergy versus traditional enthalpic terms. Assumptions associated with $\Xi(T_{Ref})$ include that: 1) $\Xi(T_{Ref})$ is only applied for the inertial process *per se*, the reactor core; 2) $\Xi(T_{Ref})$ applies only to recoverable releases; and 3) T_{Ref}, once determined, is applied consistently to all applicable NCV Method formulations (fission source and irreversible losses). The Inertial Conversion Factor and NSSS balance include:

$$\Xi(T_{Ref}) \equiv (h_{RVU} - h_{RCI}) \,/\, [g_{RVU}(T_{Ref}) - g_{RCI}(T_{Ref})] \qquad (5)$$

where:

RVU = Reactor Vessel outlet nozzle.
RCI = Core Inlet (after outer annulus losses).
h = Specific enthalpy, kJ/kg.
$g \equiv (h - h_{Ref}) - T_{Ref}(s - s_{Ref})$, Specific exergy
h_{Ref} & s_{Ref} [=] $f(T_{Ref}$, sat.liquid)

$$A_1 [\bar{\nu}_{REC}(t)]\, \Xi(T_{Ref})\, \Phi_{TH} - P_{GEN} - Q_{REJ} + B_4 m_{RV} = L_B \qquad (1ST')$$

The iterative computations resolving $\Xi(T_{Ref})$, using Eq.(1ST′) when describing a 1270 MWe PWR produced: $\Xi(T_{Ref})$ = 1.8977 at T_{Ref} = 38.290 °F (3.4944 °C). This temperature produces an average exergy in the thermal flux of 0.02384 eV, using Boltzmann's equation. This value certainly confirms the exergy of a common thermal neutron and the understanding of the inertial fission process. After serious benchmarking, $\Xi(T_{Ref})$ may well serve as the ultimate COP.

PSEUDO FUEL PIN AND AVERAGE FLUX

A final equation is developed based on Second Law modeling of a single fuel pin representing the average burn-up, pitch, flow area, etc. found in the core. This is the Pseudo Fuel Pin (PFP). Its modeling is based on: an asymmetric flux profile but whose partial axial integration produces exactly the same average flux employed in Eqs.(2ND′) and (1ST′). Asymmetry is critical, if not employed the coefficients of Eq.(2ND′) would simply repeat, with uniform scaling. Commensurate with asymmetry means a partial axial integration is required. Any PWR's symmetric trigonometric function will always produce an essentially symmetric fluid exergy gain about the core's centerline. Thus another Eq.(2ND′)-like formulation is created which solves nothing. For non-boiling reactors, changes in specific volume, viscosity, fluid velocity, etc. are simply not sufficient to effect significant fluid asymmetry. An asymmetric function for flux, $f(\Psi)$, should satisfy: i) $f(\Psi) = 0.0$ at $\Psi = b\pi$, b=0,1,2,...; ii) integrates to unity from zero to π; iii) is periodic and odd over any 2bπ; iv) $f(\Psi)$ is skewed; and v) ideally, has a non-unity peak. This is the Clausen Function of Order Two, $Cl_2(\Psi)$. $Cl_2(\Psi)$ is defined by an infinite summation, reduced using a polynomial fit with coefficients E_m, where Ψ is a function of both axial position and √Buckling, B_P, all shifted by M_T.

The fitting polynomial, normalized to exactly unity area, satisfies all functionalities. For use with the NCV Method, Ψ(y) is off-set accounting for the buckling phenomena assuming zero flux at the profile's boundaries: $\Psi[y_1 = -M_T] = 0.0$, and at: $\Psi[y_2 = 2Z + M_T] = 2(Z + M_T)B_P = \pi$.

As applied to the NCV Method's PFP, axial integration is made from the core's entrance to the point that asymmetry is most pronounced, designated as $\bar{y}$, defined as the "Differential Transfer Length" or DTL. For the PWR, the DTL is typically chosen at the Clausen's peak. For the BWR without re-circulation, asymmetry is considerably simpler, typically defined at the point DNB is reached. However, if the BWR employs re-circulation flow, then PWR methods may well apply using a mirror-image of $Cl_2(\Psi)$. Finally, the PFP equation when integrated to the DTL point, results in an unique equation versus Eqs.(2ND′) or (1ST′) ... the matrix Rank is not diminished. Note that C_E represents energy unit conversion, and r_0 is pellet radius.

$$(2D_1/B_P)[\bar{v}_{REC}(t) + \bar{v}_{LRV}(t)]\,\Phi_{TH} + D_4 m_{RV} = (2D_1/B_P)\psi_{LRV} \qquad \text{(PFP)}$$

where:
$$D_1 = C_E \pi r_0^2\, \bar{\Sigma}_F(t)\, C_{MAX\text{-}CL} \sum_{m=1}^{7} E_m [\Psi(y)]^m/m \,\Big|_{y1}^{\bar{y}} \qquad (55)$$

$$D_4 = -[g_{Core}(\bar{y}) - g_{RCI}]/ M_{FPin} \qquad (56)$$

INTEGRATION FOR AVERAGE FLUX

Eq.(PFP) as seen in Eq.(55) is defined by its peak a $C_{MAX\text{-}CL}$ (or a $C_{MAX\text{-}CO}$ for a classic cosine integration). As an aside: when converting the cosine axial peak $\Phi_{MAX\text{-}CO}$ to the average, the literature repetitiously assumes: $\Phi_{MAX\text{-}CO} = (\pi/2)\Phi_{TH}$. This is not correct. It is crucial to evaluate the <u>average</u> thermal flux associated only with the <u>active</u> core; i.e., its production of thermal power. Thus, Φ_{TH} must be evaluated as the average of the integration about the y-axis given a chopped function from -Z to +Z (not ±Z′). Such integration assumes buckling is: $B_P^2 \equiv [\pi/(2Z')]^2$, where $Z' \equiv Z + M_T$ at zero flux. For the cosine & Clausen such integrations produce:

$$\Phi_{TH} = \Phi_{MAX\text{-}CO}[(2/\pi)(1 + M_T/Z)]\,sin\,[(2/\pi)(1 + M_T/Z)]^{-1} \qquad (51)$$

$$\Phi_{TH} = \Phi_{MAX\text{-}CL}\,[(2/\pi)(1 + M_T/Z)] \sum_{m=1}^{7} E_m [\Psi(y)]^m/m \,\Big|_{y1}^{y2} \qquad (53)$$

For the common PWR, these equations become significant. Assuming a 12 foot (3.66 meter) active core with M_T taken as 6.6 cm, Eq.(51) yields for the cosine profile a $C_{MAX\text{-}CO} \approx 1.518$ (vs. the traditional π/2), see TABLE 1. Thus if ignoring Eq.(51), the computed flux would be high by 3.5%. This type of error would catastrophically bias computed electrical power, reactor coolant flow, etc. Clausen's $C_{MAX\text{-}CL}$, is computed in the same manner. Results of the average integration for Eq.(53), were taken from $y_1 = 0$ to $y_2 = 2Z$.

In summary the method demonstrated by Eqs.(51) & (53) applies to any system driven by a flux profile, provided a theoretical leakage is assumed at its physical boundaries (described by M_T), and is derived from an integratable function. It must be emphasized that NCV is not concerned with the shape of the assumed axial flux ... only that it provides critically important asymmetry and is absolutely consistent with the average flux used in Eqs.(2ND′) & (1ST′).

TABLE 1: Summary of C_{MAX}

Flux Profile	$C_{MAX} = \Phi_{MAX}/\Phi_{TH}$
Cosine, $M_T = 0.0$	π/2 = 1.57079633
Cosine with leakage	Eq.(51) => 1.51835422
Clausen, $M_T = 0.0$	Eq.(53) => 1.76589749
Clausen with leakage	Eq.(53) => 1.70603654

IRREVERSIBILITY AND FCIs

First Law conservation of flows and energies, for the closed system, is the study of conversion of enthalpic processes. In this context energy flows ($m\Delta h$) are not destroyed; they are converted. The condenser's heat rejection, and other Q losses, is what is "left" after power generation. Computing Q_{REJ} at the condenser, with other losses and with a known generation, leads directly to TC heat rate. However, the classic problem is that an accurate, direct, real-time measurement of Q_{REJ} is not credible (thus an unknown).

The Second Law is completely different, it is the study of the destruction of "available power" delivered to the system; termed G_{IN}. G_{IN} is simply all the exergy released from fission (Q_{FIS}) including antineutrino; and, we take the liberty, to include pump powers ($\sum P_{RV\text{-}k} + \sum P_{TC\text{-}k}$). Thus there are only two things that any thermal system can/will do with this G_{IN}: it will make actual brake shaft power and it will make irreversible losses. This simple approach has been used for 20 years to help monitor fossil units via the Input/Loss Method, whose thermodynamics generated a Fuel Consumption Index[2,3]. However, when applied to the nuclear system:

$$G_{IN} \equiv Q_{FIS} + \sum P_{RV\text{-}k} + \sum P_{TC\text{-}k} \qquad (31A)$$

$$= P_{GEN} + \sum I_k \qquad (31B)$$

$$1.0 = P_{GEN}/G_{IN} + \sum I_k/G_{IN} \qquad (31C)$$

where we choose to divide Eq.(31B) through by G_{IN}, and then multiply by 1000 for numerical convenience, creating Fission Consumption Indices (FCIs).

$$1000 = FCI_{Power} + \sum FCI_{Loss\text{-}k} \qquad (32)$$

Flowing from G_{IN}, FCIs are fundamentally unitless measures of the fission rate, that is its exergy flow, assigned thermodynamically to those individual components or processes responsible for the consumption of fissile material. It quantifies the exergy and power consumption of all components and processes relative to the total fission rate; by far the predominant term is the fission's recoverable exergy - as dissipated to either power or losses. For example, if the Moisture Separator Reheater's FCI increases from 200 to 210 (i.e., higher irreversible losses), which is just offset by an decrease of 10 points in FCI_{Power}, with no other changes, the operator has absolute assurance that a 5% higher portion of the fission exergy is being consumed to overcome this higher loss. This at the expense of useful brake power production; remembering we are dealing with steady state modeling. Thus recent changes to the MSR have had an adverse affect on the system, and safety ... we simply need to "understand", to be aware. FCIs will always sum to 1000, or the Second Law has been violated (e.g., bad data, un-steady state behavior, etc.).

For the nuclear system Second Law irreversible losses are defined as follows:

$$\sum I_k = \sum(1.0 - T_{Ref}/T_k)Q_{k\text{-}Loss} + Q_{NEU\text{-}Loss} + \sum(P_{ii} - m_{ii}\,\Delta g_{ii})_k - \int [mdg]_k \qquad (33)$$

Note that generator mechanical and electrical losses are, of course, irreversible but treated apart from Eq.(33).

The Second Law demands for all non-passive processes that: $\sum I_k > 0.0$. The first and second terms on the right-side of Eq.(33) represent the maximum exergy flow to the environment given a Carnot engine destruction of exergy, and the loss of unrecoverable exergy associated with an inertial process ($Q_{NEU\text{-}Loss}$). The third term represents losses due shaft inputs from pumps. Traditionally, the $\int [mdg]_k$ term represents any non-passive process having exergy exchange. For example, viable feedwater heaters in a TC, or the Steam Generator, must produce a negative exergy balance, $\int [mdg]_k$, thus an increase in irreversibility per Eq.(33); i.e., a viable heat transfer from shell to tube for a FW heater. Terms in Eq.(33) are easily computed, relying on routine measurements (given resolution of the declared unknowns).

However, the importance of Eq.(33) in understanding any inertial process becomes, perhaps, obvious. It teaches a number of situations. For example, when considering an isolated fission core, the last two terms are zero. This said, then at the instant of fission, we have no irreversible losses given: $Q_{k\text{-}Loss} = Q_{NEU\text{-}Loss} = 0.0$; noting that prompt gamma and fission fragments are recoverable. This suggests that either instant fission violates the Second Law, $\sum I_k = 0.0$; or a prompt neutrino is generated, predicted at 0.68 MeV/Fission.

VERIFICATION

Verification procedures used by the NCV Method are taken from Exergetic Systems' monitoring of fossil units, they have been in use for 20 years.[4,5] When operators pay attention ... several of our fossil installations have been in continuous use for decades. Over the years, our data analysis has been defined into two categories. The First Category encompasses data those quantities which, given a directly observable affect, exhibit a measurable outcome. Examples are pressure, temperature, quality, fluid level, and the like. Notably, mass flow, if reduced based on hydraulic principals (a valid ΔP, known nozzle condition, proper pressure taps, etc.) can be a First Category, but rarely allowed. Second Category includes that data (information) which results directly from using First Category data; e.g., we compute unit efficiency based of the laws of thermodynamics.

Important are two aspects of NCV optimization: first is that what is optimized is any parameter which one may choose provided it does not have direct impact on system integration (i.e., Second Category parameters are not allowed). These are the COPs, Λ_m. The optimization system drives differences in selected System Effect Parameters (SEPs) to zero [$\Delta\lambda_k \rightarrow 0.0$], selected as they directly impact global understanding by adjusting one or more COPs. A COP could be vessel insulation losses, miscellaneous TC losses, Steam Generator blow-down, generator losses, etc. At the procedure's core is a descriptive Objective Function which is driven to zero by changing COP values. Only resultant correction factors to COPs are transferred from the statistical routines to the thermodynamic. For example, as COPs, vessel insulation losses could be used in Eqs.(2ND) & (1ST), the equations are solved, computed generation is compared, the Objective Function is optimized, the process repeats ... nothing more. Through the years, a number of statistical analysis techniques have been implemented to drive the Objective Function. The best is a Mono Carlo technique called Simulated Annealing.[6] The Objective Function [$F(\vec{x})$] used by NCV is described by:

$$F(\vec{x}) = \sum_{k \in \mathbf{K}} \{ K - [J_0(\Delta\lambda_{GEN})]^{MC\Lambda m} - [J_0(\Delta\lambda_{FLX})]^{MC\Lambda m} - [J_0(\Delta\lambda_{RVU})]^{MC\Lambda m} - [J_0(\Delta\lambda_{FCS})]^{MC\Lambda m} - [J_0(\Delta\lambda_{FW})]^{MC\Lambda m} - [J_0(\Delta\lambda_{RV})]^{MC\Lambda m} \}_k \quad (67)$$

where:

The principal SEP is P_{GEN}, processed as:

$\Delta\lambda_{GEN} = |P_{GEN} - P_{GEN\text{-}REF}| / P_{GEN\text{-}REF}$

other SEPs may be include to aid debug and installation (flows never routinely): Φ_{TH}, g_{RVU}, $\overline{\Sigma}_F(t)$, m_{FW} & m_{RV}.

$[J_0(\Delta\lambda_m)]$ = Bessel Function, First Kind, Order Zero.

$MC_{\Lambda m}$ = Dilution Factor used for SEP sensitivity.

All COPs must carry limitations as established by physics or judgement; again, use of Second Category parameters can never be used as COPs.

RESULTS

The objective of this work was to develop viable equations which will solve for critically important NSSS unknowns. A series of different matrix techniques were used to stress Eqs.(2ND′), (1ST′), (TC′) & (PFP) solution; all indicated complete independence. The only difficulty lay with numerical values of the equation's coefficients as they vary from 10^{-9} to 10^{+10} requiring robust scaling. The unit used was Byron Unit 1 a PWR, data was obtained from the public available SAR and the author's PWR NSSS and fuel assembly design experiences. Vessel losses and pump parameters were emphasized. This included reactor vessel, steam generator, turbine casing & FW heat shell losses. Five pump classes were assumed, with estimated system pressure distributions; this included core pressure drops with grids, TC distributions, typical pump head curves, etc. Results are presented in TABLE 2. Again, the objective was not to simulate an actual Byron operational condition, but to stress the matrix solving Eqs.(2ND′), (1ST′), (TC′) & (PFP). In summary, if we can measure with accuracy electrical generation, we have intrinsically solved and verified Core Thermal Power, TC heat rate, FCIs, and more ... when using the NCV Method.

FUTURE WORK

The NCV Method offers a foundation to the industry for understanding, verifiable, system thermodynamics. It needs hard coding, review/input from PWR & BWR plant engineers, and two years of benchmarking.

NOMENCLATURE

A_1 [=] Product of units conversion, fissile volume and fission macro-cross section;

A_3, A_4, B_4, C_4 [=] Constants;

B_P^2 = Pseudo-buckling per the PFP Model, cm^{-2};

L_A, L_B, L_C [=] Loss terms, kJ/kg;

M_{FPin} = Number of fuel pins in core;

M_T^2 = Thermal neutron migration area, cm^2;

m_{RV} = Reactor vessel mass coolant flow, kg/hr;

P_{GEN} = Brake shaft generator power, kJ/hr;

P_{ii} = Shaft power delivered to pump ii; kJ/hr;

Q_{REJ} = Heat rejection at the condenser, kJ/kg;

T_{Ref} = Reference temperature per $\Xi(T_{Ref})$, ^oC or ^oK;

$\overline{\nu}_{REC}(t)$ = Recoverable fission exergy, Mev/Fission;

$\overline{\nu}_{LRV}(t)$ = Anti-neutrino fission exergy, Mev/Fission;

Φ_{TH} = Average neutron flux at criticality, $^1n\ cm^{-2}\ sec^{-1}$.

TABLE 2:
Summary Results (using Btu/hr or lbm/hr)

Parameter	Data	Data Source	Matrix Solution
Avg. Flux	1.0000×10^{13}	SAR guess	9.8438433×10^{12}
Gross Power	4.326742×10^6 1268.043MW	TC Kit	4.32674192×10^6 1268.0430 MWe
Reactor & FW Flows	138.1380×10^6 16.34750×10^6	Hand calc.	138.13800×10^6 16.34751×10^6
Heat Reject.	8.089160×10^9	TC Kit	8.0891564×10^9

REFERENCES

1. F.D. Lang, US Utility and PCT Patents Pending.
2. F.D. Lang, "Fuel Consumption Index for Proper Monitoring of Power Plants - Revised", ASME IJPGC 2002-26097, June 24-26, 2002.
3. US Patent 6799146 issued 2004, starting Col. 5.
4. US Patents 6651035 issued 2003, 6714877 issued 2004, 6810358 issued 2004, 7328132 issued 2008, 7809526 issued 2010; Canadian 2541197 issued 2011, 2754638 issued 2014; European (GB, DE, IE, CH) 1835228 issued 2010; & Australian 2006-201203 sealed 2008.
5. F.D. Lang, D.A.T. Rodgers and L.E. Mayer, "Detection of Tube Leaks and Their Location Using Input/Loss Methods", ASME Proceedings IJPGC2004-52027, 2004. ASME's Prime Movers Award.
6. W.L. Goffe, G.D. Ferrier and J. Rogers, "Global Optimization of Statistical Functions with Simulated Annealing", J. of Econometrics, Vol.60, No.1 of 2, pp.65-100, Jan./Feb. 1994.